Minisymposium on Billiards

All talks are in room CC012 James France Building. Each talk is 25 minutes (not including time for questions).

Tuesday 10:30-12:30: Reflections

Jose Pedro Gaivao: Dynamics of pinball billiards

In this talk I will consider a special class of polygonal billiards with modified reflection laws: pinball billiards. A pinball billiard has a reflection law that contracts the angle of reflection by a constant factor towards the normal of the table. These billiards no longer preserve the Liouville measure and may have hyperbolic attractors. I will survey recent results concerning the existence of SRB measures and their ergodic properties.

John Smilie: TBA

Gianluigi DelMagno: TBA

Arnd Bäcker: 3D Billiards: visualization of the 4D phase space and power-law trapping of chaotic trajectories

Understanding the transport properties of higher-dimensional systems is of great importance in a wide variety of applications, e.g., for celestial mechanics, particle accelerators, or the dynamics of atoms and molecules. A prototypical class of model systems are billiards for which a Poincare section leads to discrete-time map. For the dynamics in three-dimensional billiards a four-dimensional symplectic map is obtained which is challenging to visualize. By means of the recently introduced 3D phase-space slices [2] an intuitive representation of the organization of the mixed phase space with regular and chaotic dynamics is obtained [1]. Of particular interest for applications are constraints to classical transport between different regions of phase space which manifest in the statistics of Poincare recurrence times. For a 3D paraboloid billiard we observe a slow power-law decay caused by long-trapped trajectories which we analyze in phase space and in frequency space. Consistent with previous results for 4D maps [3] we find that: (i) Trapping takes place close to regular structures outside the Arnold web. (ii) Trapping is not due to a generalized island-around-island hierarchy. (iii) The dynamics of sticky orbits is governed by resonance channels which extend far into the chaotic sea. We find clear signatures of partial transport barriers. Moreover, we visualize the geometry of stochastic layers in resonance channels explored by sticky orbits.

[1] 3D Billiards: Visualization of Regular Structures and Trapping of Chaotic Trajectories M. Firmbach, S. Lange, R. Ketzmerick, and A. Bäcker, arXiv:1805.06823 [nlin.CD] (2018).

[2] Visualization and comparison of classical structures and quantum states of four-dimensional maps, M. Richter, S. Lange, A. Bäcker, and R. Ketzmerick, Phys. Rev. E 89, 022902 (2014).

[3] What is the mechanism of power-law distributed Poincare recurrences in higher-dimensional systems? S. Lange, A. Bäcker, and R. Ketzmerick, Eur. Phys. Lett. 116 30002 (2016).

[4] For videos of 3d phase space slices see: https://wwwpub.zih.tu-dresden.de/ baecker/supp

Tuesday 15:30-17:30: Time-dependent billiards

Crt Lozej: Momentum diffusion in the stadium billiard

In this talk I will present some recent results on the momentum diffusion in the stadium billiard introduced by Bunimovich. The stadium billiard is rigorously proven to be ergodic and mixing. I will show that the results for the diffusion in momentum space obtained by numerical calculations of the stadium dynamics agree very well with an inhomogeneous diffusion equation. The diffusion constant is a parabolic function of the canonical momentum. The model enables us to extract the classical transport time, an important parameter in the study of localization of chaotic eigenstates in the quantum stadium billiard. Lastly, I will compare the momentum diffusion in the stadium with the momentum diffusion in a family of billiards with divided phase space.

Julian Talbot: TBA

André Livorati: Investigation of Fermi acceleration in a bouncing ball system: Stickiness influence and transition from normal to super diffusion

Some phase space transport properties for a bouncing ball system are studied. The system is composed of a particle, or an ensemble of non-interacting particles experiencing elastic collisions with a heavy and periodically moving wall under the influence of a constant gravitational field. The dynamics lead to a mixed phase space where chaotic orbits have a free path to move along the velocity axis, presenting then a normal diffusion behavior. Depending on the control parameter, that is physically interpreted as a ration between accelerations, we may observe stickiness influence in the transport of chaotic orbits that generates a slowing mechanism for Fermi acceleration, as well as the presence of featured resonances, known as accelerator modes, that lead to a ballistic growth of velocity.

Giulio Casati: Exponential localisation in right triangular billiards

We show evidence, based on extensive and carefully performed numerical experiments, that the system of two elastic hard-point masses in one dimension is not ergodic for a generic mass ratio and consequently does not follow the principle of energy equipartition. This system is equivalent to a right triangular billiard. Remarkably, following the time-dependent probability distribution in a suitably chosen velocity direction space, we find evidence of exponential localization of invariant measure. For non generic mass ratios which correspond to billiard angles which are rational, or weak irrational multiples of π, the system is ergodic, consistent with existing rigorous results. We also introduce a new family of billiards which break time reversal symmetry in spite of having piece - wise straight trajectories. We show that our billiards preserve the ergodic and mixing properties of conventional billiards.

Wednesday 10:30-12:30: Waves/experiments

George Datseris: Applying Kac's Lemma into solid-state nanodevices through billiards

In this talk we are considering antidot-superlattices, real nanodevices that can be approximated by periodic Sinai billiards. Experiments in these devices unexpectedly observe ballistic features beyond the mean free time. We find that the reason of these observations is the structure of the mean collision time in the periodic Sinai billiard, which can be connected with the experimentally measured quantity, the resistance of the device.

The structure of the mean collision time is very peculiar and directly proportional to the available phase-space portion in the periodic Sinai billiard with perpendicular magnetic field. Using Kac's lemma, a fundamental property of discrete systems that conserve volumes, we are able to prove the numerically observed relation.

The connection we observe and prove is a general mechanism that can allow observations of ballistic features beyond the mean free time in a wide range of mesoscopic systems, where available phase-space volumes can be varied through a change in an external parameter.

Andre Brandstötter: Coherent wave scattering in billiards: time-delay and beyond

We present a protocol for manipulating waves inside billiards or waveguides based on the Wigner-Smith time-delay operator. The concept of this operator emerged in scattering theory as a very useful tool to deduce the time associated with a scattering event from stationary measurements of the corresponding scattering amplitudes. Originally devised for nuclear scattering problems [1,2], this concept later resurfaced in mesoscopic physics [3]. The Wigner-Smith time-delay operator Q is constructed based on a system's scattering matrix S by way of a frequency derivative, $Q = -iS^{-1}\frac{\partial S}{\partial \omega}$. The eigenvalues of Q, also called proper delay times, measure the time-delay caused by the scattering process at a given potential and their average value just depends on the outer shape of a billiard, but not its inner structure [4]. The corresponding eigenvectors of Q are states that can be associated with this well-defined time-delay. A slight variation of these states input frequency leaves the output profile invariant to first order - a very attractive property for light transmission through multi-mode fibers [5]. In the ballistic limit of vanishing disorder, this property leads to particle-like wave function patterns [6] even in billiards with chaotic classical dynamics [7]. In our recent work we took the concept of time-delay eigenstates to a new level such as to produce states that instead of being insensitive with respect to a frequency variation are invariant with respect to changes in the system configuration, like a local shift of a designated scatterer inside a billiard or a disordered medium [8]. In the same way as the frequency-insensitive principal modes are the eigenstates of the time-delay operator Q (involving a frequency derivative), the states we are looking for are eigenstates of a corresponding operator $Q_{\alpha=} - iS^{-1}\frac{\partial S}{\partial \alpha}$, where the parameter α stands, e.g., for the position of a movable scatterer. Depending on the choice of the parameter α, we can either focus onto a predetermined target inside a billiard or apply a well-defined torque onto it [9].

[1] E. P. Wigner, Lower Limit for the Energy Derivative of the Scattering Phase Shift, Phys. Rev., vol. 98, no. 1, pp. 145147, Apr. 1955.

[2] F. T. Smith, Lifetime Matrix in Collision Theory, Phys. Rev., vol. 118, no. 1, pp. 349356, Apr. 1960.

[3] P. W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Quantum Mechanical Time-Delay Matrix in Chaotic Scattering, Phys. Rev. Lett., vol. 78, no. 25, pp. 47374740, Jun. 1997.

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[5] W. Xiong, P. Ambichl, Y. Bromberg, B. Redding, S. Rotter, and H. Cao, Spatiotemporal Control of Light Transmission through a Multimode Fiber with Strong Mode Coupling, Phys. Rev. Lett., vol. 117, p. 053901, Jul. 2016.

[6] S. Rotter, P. Ambichl, and F. Libisch, Generating Particlelike Scattering States in Wave Transport, Phys. Rev. Lett., vol. 106, no. 12, p. 120602, Mar. 2011.

[7] J. Böhm, A. Brandstötter, P. Ambichl, S. Rotter, and Ulrich Kuhl, In situ realization of particlelike scattering states in a microwave cavity, Phys. Rev. A 97, p. 021801 (R), Feb. 2018.

[8] P. Ambichl, A. Brandstötter, J. Böhm, M. Khmayer, U. Kuhl, and S. Rotter, Focusing inside Disordered Media with the Generalized Wigner-Smith Operator, Phys. Rev. Lett., vol. 119, no. 3, p. 033903, Jul. 2017.

[9] M. Horodynski, M. Kühmayer, A. Brandstötter, and S. Rotter, Manuscript in preparation.

Henning Schomerus: From copropagating modes to complex dispersion arcs in nonhermitian resonator arrays

Copropagating pairs of modes are a common feature of deformed microdisk cavities that lack mirror symmetries [1]. Here I explore the consequences when these mods are distributed across resonator arrays. This leads to transport effects mimicking PT symmetry [2], the apparance of interface states in topological spectral phase transitions [3], and the formation of complex bulk and edge-state arcs in two dimensional arrays [4]. All these effects are intimately linked to the apearance of exceptional points.

[1] Nonorthogonal pairs of copropagating optical modes in deformed microdisk cavities J. Wiersig, A. Eberspcher, J.-B. Shim, J.-W. Ryu, S. Shinohara, M. Hentschel, and H. Schomerus Phys. Rev. A 84, 023845 (2011)

[2] Nonhermitian transport effects in coupled-resonator optical waveguide Henning Schomerus and Jan Wiersig Phys. Rev. A 90, 053819 (2014)

[3] Topologically protected defect states in open photonic systems with non-hermitian charge-conjugation and parity-time symmetry S. Malzard, C. Poli, H. Schomerus Phys. Rev. Lett. 115, 200402 (2015)

[4] Bulk and edge-state arcs in non-hermitian coupled-resonator arrays S. Malzard and H. Schomerus arXiv:1805.08161 [physics.optics] submitted (2018).

Michał Ławniczak: Experimental investigation of short- and long-range spectral fluctuations in microwave networks and billiards

We present the experimental study of short- and long-range spectral fluctuations in microwave networks and billiards. A special attention will be paid to missing level statistics of microwave networks and three-dimensional chaotic microwave cavities. In this case the investigation is reinforced by the power spectrum of level fluctuations analysis which also takes into account missing levels [2, 1, 3, 4]. On the basis of our data sets we demonstrate that the power spectrum of level fluctuations in combination with short- and long-range spectral fluctuations provides a powerful tool for the determination of the fraction of randomly missing levels in systems that display wave chaos. We also show that in the case of incomplete spectra with many unresolved states forming clusters the above procedures may fail. In such a case the random matrix theory calculations can be applied for the determination of the fraction of missing levels.

This work was supported in part by the National Science Centre Grant No. 2016/23/B/ST2/03979.

[This is joint work with Vitalii Yunko, Małgorzata Białous, Szymon Bauch, Barbara Dietz, and Leszek Sirko]

References

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[3] M. Ławniczak, M. Białous, V. Yunko, S. Bauch, B. Dietz, and L. Sirko, Acta Phys. Pol. A **132**, 1672 (2017).

[4] M. Ławniczak, M. Białous, V. Yunko, S. Bauch, and L. Sirko, Phys. Rev. E **98**, 012206 (2018).

Thursday 10:30-12:00: Hyperbolic billiards

Péter Bálint: Polynomial decay of correlations for billiard flows

I plan to describe a method to obtain upper bounds on the decay of correlations for a class of non-uniformly hyperbolic flows, including some important billiard examples. In particular this way it is established that the decay rate is $O(t^{-1})$ for the flows in planar dispersing billiard domains with infinite horizon and in Bunimovich stadia. This is joint work with Oliver Butterley and Ian Melbourne.

Benoît Saussol: Spatio-temporal Poisson process for hitting times

For many measure preserving dynamical systems the successive hitting times to a small set is well approximated by a Poisson process on the real line. In this work we define a new process obtained from recording not only the successive times of visits to a set A, but also the position in A of the orbit at visit times, in the limit where the measure of A goes to 0. We obtain a convergence of this process, suitably normalized, to a Poisson point process in time and space under some decorrelation condition. We present several new applications to hyperbolic maps and SRB measures, including the case of a neighborhood of a periodic point, and some billiards such as Sinai billiards, Bunimovich stadium and diamond billiard. [Joint work with Francoise Pene]

Domokos Szász: Fourier law from Hamiltonian dynamics

For deriving Fourier law from Hamiltonian dynamics in 2008 Gaspard and Gilbert came up with a billiard model and suggested a two step approach for it: 1. for the energies of the particles derive in the rare interaction limit a Markov jump process (dynamical part); 2. take the hydrodynamic limit of the obtained jump process; this is expected to indeed lead to the Laplace equation (stochastic part). Since their model was still unsuitable for mathematics, we first introduced its tractable variant: the disk-piston model. For it we can show that its rare interaction limit is, indeed, a Markov jump process. The talk is based on joint works with P. Balint, Th. Gilbert, P. Nandori, IP. Toth.

Thursday 15:30-17:00: Transport I

Thomas Gilbert: Nonlinear stability of the periodic orbits of four-dimensional symplectic billiard maps

The periodic orbits of three-dimensional cylindrical stadium billiards typically combine a pair of elliptic or hyperbolic eigenvalues with a parabolic one, whose stability cannot be decided based on linear perturbation theory alone. By invoking a separation of time scales between the fast evolution of perturbations along the elliptic/hyperbolic plane and the slow evolution along the parabolic one, I will show that averaging techniques can be introduced which allow to characterise the nonlinear stability of these orbits. A particular instance of such orbits are whispering gallery modes spiralling about the cylindrical cavity, which thus avoid dispersion.

Leonid Bunimovich: Physical versus mathematical billiards

In physical billiards a hard ball of radius r moves in a billiard table. In standard mathematical billiards it is a point particle, i.e. $r = 0$. Despite a general opinion that both billiards have similar dynamics I will demonstrate that anything can happen in another. Namely chaos order transition may appear softly (for any $r > 0$) transition from one to another. Namely chaos order transition may appear softly (for any $r > 0$) or in a hard way (when r becomes greater some positive critical value). Besides the physical Ehrenfests' Wind Tree model neglected in Statistical Mechanics is dynamically richer than a favorite of Statistical Mechanics Lorentz gas.

Carlos Mejia-Monasterio: TBA

Friday 10:30-12:00: Transport II

Marco Lenci: Global-local mixing for extended chaotic dynamical systems

I will recall the notion of global-local mixing for a dynamical system preserving an infinite measure and prove that it holds for a class of simple extended systems, namely, Z-invariant expanding Markov maps of the real line and their finite modifications. Then I will illustrate how global-local mixing can be used to derive certain statistical properties of global observables, i.e., functions that are supposed to measure non-local quantities in phase space. These ideas apply to more complicated systems as well, such as periodic Lorentz gases, etc.

Chris Lutsko: The Lorentz gas: going beyond the Boltzmann-Grad limit

We use coupling methods to prove a central limit theorem for the Lorentz process in a random scattering configuration in the low density limit as time is taken to infinity. It has long been known that the random Lorentz process in the low-density limit converges (in several senses) to a random flight process for a finite time. We use this connection to motivate coupling the two processes in order to prove a central limit theorem for the Lorentz process in infinite time. This is joint work with Blint Toth.

Françoise Pène: Stochastic properties of the Z^2-periodic Sinai billiard

The Z^2-periodic Sinai billiard is a famous example of chaotic dynamical system preserving an infinite measure. The goal of this talk is to present some of its properties such as: recurrence, ergodicity, mixing, limit theorems. A crucial point in this study is the representation of this system by a Z^2-extension of a chaotic dynamical system: the Sinai billiard in the torus. We will compare results for the Sinai billiard in the torus and for the Z^2-periodic Sinai billiard. We will see the link between the stochastic properties of the Z^2-periodic Sinai billiards and the limit theorems (namely the local limit theorem) for the Sinai billiard in the torus.