Dynamics of Localized Structures in Nonlinear Wave Evolutions

Organisers: V.M.Rothos, H. Susanto, N. Smyth

The minisymposium will bring together specialists studying nonlinear waves in PDEs and nonlinear lattices. The particular emphasis will be given to recent results of single and multi-component nonlinear wave-type and nonlinear lattice equations with local and/or nonlocal terms as well as orbital and asymptotic stability of breathers and gap solitons, multi-solitons. This minisymposium touches, via a diverse cohort of experts, upon the current state-of-the-art in this field and the challenges that lie ahead. A balanced perspective encompassing theory, computation and experiment will be sought that should be of value to newcomers, as well as to seasoned researchers in the field.

List of Speakers

1. S. Roy Choudhury, University of Central Florida, USA
2. Guillaume JAMES Grenoble INP - Ensimag , France
3. Simone Paleari, Univeristy of Milan, Italy
4. Nikolaos Efremidis, University of Crete, Greece
5. Vassilis M Rothos, Aristotle University of Thessaloniki, Greece
6. N. Smyth, University of Edinburgh, UK
7. Jonathan Wattis, Nottingham University, UK
8. Karima Khusnutdinova, Loughboorugh University, UK
9. Matthew Tranter,Loughboorugh University, UK
10. Rahmi Rusin, University of Essex,UK and Universitas Indonesia
11. Matteo Sommacal, University of Northumbria, UK
12. V. V. Konotop, Universidade de Lisboa, Portugal
13. Jaime Cisternas, Universidad de los Andes, Santiago, Chile
14. Heather Nelson, University of Oxford, UK

Various Dynamical Regimes, and Transitions from Homogeneous to Inhomogeneous Steady States in Delayed Oscillators under Different Types of Coupling

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This talk will involve coupled oscillators with multiple delays, and dynamic phenomena including synchronization at large coupling, and a variety of behaviors in other parameter ranges including transitions between Amplitude Death (homogeneous steady states) and Oscillation Death (inhomogeneous steady states). Both analytic (multiple scales/energy methods) and numerical results would be presented. Behaviours in both limit cycle and chaotic oscillators will be compared in both cyclic and diffusive coupling scenarios.

Localized waves in the discrete p-Schrödinger equation

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We study the existence of localized waves in the discrete p-Schrödinger (DpS) equation. This model consists of a one-dimensional discrete nonlinear Schr "odinger equation with strongly nonlinear inter-site coupling (a discrete p-Laplacian). The DpS equation describes the slow modulation in time of small amplitude oscillations in different types of nonlinear lattices, where linear oscillators are coupled to nearest-neighbors by strong nonlinearities. Such systems include granular chains made of discrete elements interacting through a Hertzian potential ($p = 5/2$ for contacting spheres), with additional local potentials or resonators inducing local oscillations. For p lying slightly above 2, we derive different continuum approximations to the DpS equation in order to approximate traveling breather solutions. One model is the logarithmic nonlinear Schrödinger (NLS) equation which admits Gaussian solutions, and the other are fully nonlinear degenerate NLS equations with compacton solutions. Moreover, in an opposite limit where the parameter p becomes large (vibroimpact limit), we compute an analytical approximation of solitary wave solutions of the DpS equation.

On the (non)existence of degenerate phase-shift localised solution in dNLS and KG nonlocal lattices

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We investigate the existence of four-site discrete solitons (multibreathers) in dNLS (KG) onedimensional lattices featuring interactions beyond nearest neighbors, in the small coupling regime; such solutions are reminiscent of two-dimensional vortex-like waveforms. Due to the nondegeneracies of such solutions, standard continuations techniques do not apply. We overcome such difficulties using a combination of perturbation methods to exploit the existence of a conserved density current, and a Lyapunov-Schmidt decomposition technique.

Nonlinear imaging in photonic lattices

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We use the Floquet spectrum of z-dependent lattices to engineer the coupling coefficients in periodic lattices. We show that nonlinear imaging is possible in periodic waveguide configurations, provided that we use two different segments of nonlinear media with opposite signs of the Kerr nonlinearity with, in general, no other restriction about their magnitudes. The second medium is used to implement effective reverse propagation. In this respect, a main ingredient in achieving nonlinear imaging is the control of the sign and the amplitude of the coupling coefficient. We numerically test our results in one- and two-dimensional square arrangements of waveguides.

Stability of gap soliton in the present of a weak nonlocality in periodic potentials

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In this talk, we study the stability and internal modes of one-dimensional gap soliton employing the modified nonlinear Schrödinger equation with a sinusoidal potential together with the present of a weak nonlocality. Using an analytical theory, it is proved that two soliton families bifurcate out from every Bloch-band edge under self-focusing or self-defocusing nonlinearity, and one of these is always unstable. Also, we study the oscillatory instabilities and internal modes of the modified nonlinear Schrödinger equation. The analytical results are in excellent agreement with numerical results.

Solitons in spin-orbit coupled Bose-Einstein condensates with periodically varying parameters

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We report on the existence and properties of solitons in spin-orbit coupled Bose-Einstein condensates with periodic parameters. These are vector (spinor) solitons obeying the internal degree of freedom (pseudo-spin). The main attention is paid to the gap solitons in Zeeman lattices, i.e. to the periodic potentials having opposite signs for two spinor components of the condensate, as well as to the periodically varying spin-orbit coupling. It is shown that in the latter case the problem is reduced to the integrable Manakov system. Adding a constant Zeeman splitting, which results in the frequency mismatch of the decoupled components of the SO-BEC spinor, significantly changes soliton properties. In particular, this brakes the integrability and leads to nontrivial soliton collisions.

Breathers and Pulses in general Fermi-Pasta-Ulam and Klein-Gordon lattices

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We consider a variety of general lattice systems governed by Klein-Gordon, and/or Fermi-Pasta-Ulam interactions in one- and two-dimensions, and either with all nodes identical, or diatomic chains, in which light and heavy masses alternate. We show how asymptotic methods can be generalised to systems in which multiple components of the solution need to be determined at each order of the asymptotic series. As an example, we consider travelling waves in the one-dimensional diatomic FPU lattice (which includes the Toda lattice) in which (i) find the leading order shape of the travelling pulse; (ii) find correction terms, some of which simply generate phase shifts of the pulse, whilst other terms explain the development of internal oscillations in the pulse which, in numerical simulations, have been found to develop. We also consider the approximation of breathers in two-dimensional monatomic lattices, in which each node can be displaced in both of the directions defined by the place of the lattice; hence the displacement has two components, with governing equations which couple them together.

Resonant Korteweg-de Vries dispersive shock waves and Whitham modulation theory

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Dispersive shock waves, termed undular bores in fluid applications, are a widespread nonlinear dispersive wave phenomenon with applications in areas including water waves, oceanography, meteorology and optics. Dispersive shock waves are an unsteady modulated wave form, which in standard form consist of solitary waves at one edge and linear waves at the opposite. The standard technique to find dispersive shock wave solutions of nonlinear dispersive wave equations is Whitham modulation theory, a powerful technique used to analyse slowly varying wavetrains. When these equations are hyperbolic, dispersive shock waves are found as centred simple wave solutions. When higher order dispersive corrections are added to generic nonlinear dispersive wave equations, such as the Korteweg-de Vries (KdV) and nonlinear Schrödinger (NLS) equations, the standard dispersive shock wave structure undergoes dramatic changes due to wave resonances. This talk will consider the KdV equation with fifth order dispersion as an example of this phenomenon. The various types of new dispersive shock wave structures for this equation will be discussed. Whitham modulation theory will be used to find the new solution for a bore structure which does not occur for the standard KdV equation, based on the concept of a partial undular bore which does not have a solitary wave edge.

Detecting delamination with the help of solitons

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It has been shown in experiments that long longitudinal bulk strain solitary waves can propagate in elastic materials such as PMMA and polystyrene for long distances without significant decay in their amplitudes, in contrast to linear waves. This makes them an attractive candidate for introscopy. The behaviour of longitudinal waves in the delaminated regions of a layered structure are of particular interest, as any changes in their structure may allow us to detect the presence of delamination. We consider two cases: a bar with a perfect bond between the layers or a sufficiently soft ("imperfect bond") between the layers. Pure solitons (perfect bonding) or radiating solitary waves (soft bonding) in these structures have been observed in experiments.

We model the propagation of longitudinal waves in each case, using a direct and semi-analytical (weakly-nonlinear) numerical approaches. For the perfectly bonded bar, we confirm the theoretical result that the incident soliton fissions into multiple solitons in the delaminated region of the bar, where the number of secondary solitons is dependent upon the geometry of the waveguide. The fission of an incident soliton has been observed in experiments. Theoretical estimates are obtained using the Inverse Scattering Transform, and these are in good agreement with the numerical results. We further consider the case of a finite delamination and show that changes in the amplitude of the lead soliton can be used to estimate the length of the delamination.

For the two-layered bar with a soft bonding layer, supporting a radiating solitary wave, we show that the solitary wave separates from its radiating tail in the delaminated region of the bar, a clear sign of delamination. We also show that, for a finite delamination, the delamination length can be inferred from the change in amplitude. The structure of the wave in the second bonded section also suggests the relative position of the delamination.

Weakly-nonlinear approach to initial-value and scattering problems for Boussinesq-type equations

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Boussinesq-type equations have recently emerged in many problems describing nonlinear waves in solids, including nonlinear longitudinal bulk strain waves in layered elastic waveguides. Dynamical behaviour of a layered structure depends not only on the properties of the bulk material but also on the type of bonding between the layers. Pure solitary waves can propagate in a perfectly bonded layered structure, but if the layers have similar longitudinal wave speeds and the bonding between the layers is sufficiently soft ("imperfect bonding"), then a pure soliton is replaced with a radiating solitary wave, i.e. a solitary wave with a co-propagating oscillatory tail. We are concerned with the construction of weakly-nonlinear solutions of the initial-value and scattering problems for Boussinesq-type equations. The mathematical problem formulation of the scattering problem consists of a set of Boussinesq-type equations with piecewise-constant coefficients for longitudinal displacements (differing in different sections of the structure), subject to continuity of longitudinal displacement and normal stress at the interfaces between the sections. Our approach is based on the use of asymptotic multiple-scale expansions and averaging with respect to fast variables, leading either to Korteweg-de Vries or Ostrovsky-type equations in various sections of the structure (to leading order). Partial theoretical estimates are obtained using the Inverse Scattering Transform. The weakly-nonlinear approach agrees well with direct numerical simulations.

This is joint work with A.M. Samsonov, K.R. Moore, D.E. Pelinovsky and M.R. Tranter.

False instability and its remedy in the application of variational method in discrete nonlinear Schrödinger equations

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We study discrete solitons of the discrete nonlinear Schrödinger equation and their stability via a variational method using Gaussian ansatz. We report a novel observation in the presence of a false instability where the result is confirmed with numerical computation. Comparing with established results and using Vakhitov-Kolokolov criterion, we deduce that the instabilities are due to the tail error. We also obtain a remedy for the false instability by employing multiple Gaussian functions as our ansatz. We show that the higher the number of Gaussian function used, the better the approximations of the solutions.

Integrability, modulation instability and the onset of rational solitons for nonlinear wave equations

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Recently, a direct construction of the eigenmodes of the linearization of 1+1 multicomponent, nonlinear, partial differential equations of integrable type has been introduced. This construction employs only the associated Lax pair, with no reference to spectral data and boundary conditions. In particular, this technique allows to study the instabilities of continuous wave solutions in the parameter space of their amplitudes and wave numbers, as well as to compute and potentially classify the so-called stability spectra. In this context, it provides a necessary condition in the parameters for the onset of rational solitons from modulation instability. The theory will be presented using the example of a system of two coupled nonlinear Schrdinger equations in the defocusing, focusing and mixed regimes. If time will allow, different applications of this theory will be illustrated, including the resonant interaction of three waves.

Normal and anomalous diffusion of two-dimensional solitons

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The dissipative solitons that appear in nonlinear optics can suffer explosions: after remaining localized for a certain period of time without major profile changes, they can grow and become broader for a short time, returning to the original spatial profile afterwards, and repeating the cycle intermittently. We consider the dynamics of dissipative solitons in two spatial dimensions, inspired by a model of mode-locked lasers based in the complex Ginzburg-Landau equation. We found that there are two regimes for the spatial motion of the soliton: in the normal regime the soliton explodes asymmetrically most of the time; and in the subdiffusive regime the soliton experiences long sequences of symmetric explosions before exploding asymmetrically. We analyzed the solitons trajectories in both regimes using the tools of anomalous diffusion. For the normal regime we found statistics similar to Brownian motion. For the subdiffusive regime we observed large trajectory-to-trajectory variations and weak ergodicity breaking that can be explained using a simple continuous-time random walk model. In this presentation we analyze the distributions of generalized diffusivities for the trajectories, and the distributions of relative angles, that explain the transition between the regimes and the destructive effect of additive noise.

Variability in Fermi, Pasta and Ulam Lattices

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The recurrence phenomenon was first observed by Fermi, Pasta, Ulam and Tsingou in numerical experiments in 1955. Since then, recurrence has only been observed in very limited practical applications. We believe that this is due to real world conditions that are not considered in the original theoretical experiments. We report on the impact of such conditions on the FPUT recurrence phenomenon, and explore the limits of these conditions under which we believe that recurrence could still be observed. We show that variability in the oscillators due to manufacturing tolerances can degrade the observance of recurrence. The number of oscillators in the chain affects the tolerance that can be absorbed by the system before recurrence breaks down. A short chain of 8 oscillators shows recurrence at a tolerance of 10