Open quantum systems and the Hörmander condition

Roman Shubert (Bristol University)

Abstract

This joint work with J. Parsons and T. Plastow.
An open quantum systems is a quantum system coupled to an environment and its dynamics can be described by the Lindblad equation. The phase space representation of the Lindblad equation is a diffusive equation in the sum of squares form of Hoermander, and one of the main physical effects of the diffusive terms is decoherence. We will give a brief introduction into decoherence and describe how techniques developed to study hypoellipticity, in particular the Hoermander condition, can be used to study and quantify decoherence. We will mainly focus on a simple class of systems studied first by Lanconelli and Polidoro, but give as well a brief outlook on connections to sub-Riemannian geometry and analysis on Carnot groups.

Very weak solutions of hyperbolic equations with multiplicities and singular coefficients

Claudia Garetto (Loughborough University)

Abstract

In this joint work with Michael Ruzhansky (Imperial College) we study the well-posedness of the Cauchy problem for hyperbolic equations with multiplicities and highly singular coefficients. A notion of very weak solution is introduced and its relation with the classical solution (when it exists) are investigated.

Microlocal analysis for hyperbolic systems – Old and new

Christian Jäh (Loughborough University)

Abstract

We consider the Cauchy problem

$$\begin{cases} D_t u = A(t,x,D_x)u + B(t,x,D_x)u + f(t,x), \ (t,x) \in [0,T] \times \mathbb{R}^n \\ u(0,x) = u_0(x), \ x \in \mathbb{R}^n, \end{cases}$$

with $A(t,x,D_x)$ and $B(t,x,D_x)$ being $m \times m$ matrices of pseudo-differential operators of class $C^0([0,T],\Psi^1_{1,0})$ and $C^0([0,T],\Psi^0_{1,0})$, respectively. Further it is assumed

that the system is hyperbolic, i.e. that the eigenvalues of $A(t, x, \xi)$ are all real.
In this talk, we give a brief overview on results on systems with variable multiplicities and discuss some new developments in the special case of upper triangular systems. We discuss in particular, how the lower order terms influence the well-posedness and propagation of singularities.

References

[1] Claudia Garetto and Christian Jäh and Michael Ruzhansky *Hyperbolic systems with non-diagonalisable principal part and variable multiplicities, I: well-posedness* Mathematische Annalen, 2018, doi:10.1007/s00208-018-1672-1.

[2] Claudia Garetto and Christian Jäh *Well-posedness of hyperbolic systems with multiplicities and smooth coefficients* Mathematische Annalen, 369(1–2), pp. 441485 (2017), doi:10.1007/s00208-016-1436-8.

[3] Ilia Kamotski and Michael Ruzhansky *Regularity properties, representation of solutions and spectral asymptotics of systems with multiplicities*, Comm. Partial Differential Equations, 32 (2007), pp. 1-35, doi:10.1080/03605300600856816.

The inverse spectral transform for the conservative Camassa-Holm flow

Jonathan Eckhardt (Loughborough University/Vienna University)

Abstract

The Camassa-Holm equation is a nonlinear partial differential equation that models unidirectional wave propagation on shallow water. I will show how to integrate this equation by means of solving an inverse spectral problem for a Sturm-Liouville problem with an indefinite weight. The global conservative solutions obtained in this way form into a train of (in general infinitely many) peakons in the long-time limit.