Abstracts of the Minisymposium "Invariant sets in dynamical systems"

Conference "Dynamics Days Europe", 3-7 September 2018

Organizers of the Minisymposium:

Prof. Alexander Formalskii (Lomonosov MSU, Moscow),

Prof. Alois Steindl (TU Wien, Vienna),

Dr. Liubov Klimina (Lomonosov MSU, Moscow).

Subsessions: Invariant sets and manifolds in systems with control;

Methods of searching for invariant sets in dynamical systems;

Bifurcations of invariant sets in dynamical systems.

Introduction of the Minisymposium: A wide range of fundamental and applied mechanical problems can be studied in the paradigm of dynamical systems. Attracting invariant sets in such systems usually correspond to available steady modes of functioning of mechanical, electromechanical, biomechanical systems. Such modes can be either desirable operation regimes or modes that are to be avoided. A corresponding range of tasks arises: such as to describe bifurcations and stability of invariant sets; to construct control strategies allowing stabilization of unstable invariant sets; to describe the domain of attraction of "preferable" invariant sets and to enlarge this domain using control, etcetera.

Stable and unstable invariant manifolds of the canonical equations also play an important role for optimal control problems on semi-infinite and large time intervals.

Corresponding fundamental approaches and applied problems are welcomed to be discussed in the present minisymposium "Invariant sets in dynamical systems" in the frame of the "Dynamics Days Europe" Conference.

List of abstracts

Invariant sets and manifolds in systems with control

Methods of searching for invariant sets in dynamical systems

Bifurcations of invariant sets in dynamical systems

Stabilization of a tethered satellite system by tension control

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The final stage of the deployment process of a tethered satellite system is investigated: When the sub-satellite is already close to its target position, the swinging motions of the satellite and the tether oscillations should be damped out by applying a proper tension force on the tether at the outlet point. While it is possible to reach the final state in finite time, if the motion remains in the orbital plane, the tension control acts parametrically for out-of-plane oscillations and the corresponding oscillation can be extinguished only algebraically slowly. For a massless tether it was shown, that the dynamics is governed by a Hamiltonian Hopf bifurcation and the optimal control is determined by the stable manifold. Computing the Normal Form a control law for both in-plane and out-of-plane oscillations can be found. In this talk that approach is applied to a tethered satellite system with a light tether.

Ball on the beam under saturated control: stabilization with large basin of attraction

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We consider here the problem of stabilization of a ball that can roll without slipping on the *straight* (see Fig. 1) or *curvilinear* (see Fig. 2) beam. The beam may turn about its pivot point *O* that is located below it. Thus the beam is similar to an inverted pendulum. In the equilibrium the beam is located horizontally, and the ball is in the middle *A* of the beam. Torque *L* developed by an electric DC motor is applied in the pivot. The considered system has two degrees of freedom and it is controlled by only single torque *L*, thus it is under-actuated. Angles φ and θ are generalized coordinates of the system. Voltage *u* supplied to the motor is assumed limited in absolute value: $|u| \le u_0 \quad (u_0 = const)$. The system has an unstable (when no control torque is applied) equilibrium that is to be stabilized by means of the motor. In the case of the *straight* beam, the linearized near unstable equilibrium system has *one* positive eigenvalue; if the curvature of the *curvilinear* beam is sufficiently large, then the corresponding system linearized near unstable equilibrium has *two* positive eigenvalues; all the other eigenvalues have negative real parts. Thus, the system with the curvilinear beam is more difficult to stabilize than with straight one.

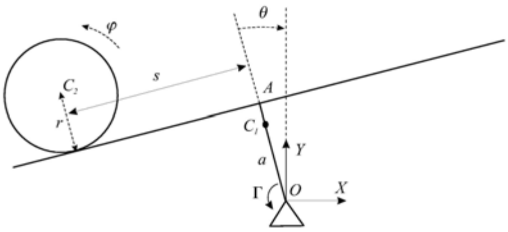

Figure 1.

Using a non-degenerate linear transformation, linearized system can be reduced to Jordan form. After we separate unstable modes from this system in Jordan form – for the ball on the straight beam it is only one mode, for the ball on the curvilinear beam there are two unstable modes. We design control law using "unstable variables" in the feedback loop taking into account saturations imposed on the control signal. By this way a large basin of attraction of the desired equilibrium can be ensured. So, if we want to ensure large basin of attraction of the desired equilibrium it is necessary to use all resources of control for suppressing the *unstable* modes. When we suppress unstable mode for the *straight* beam (see Fig. 1), the basin of attraction coincides with the controllability domain [1]. And it is maximal as possible basin of attraction for this case. If we suppress the both unstable modes for the ball on the *curvilinear* beam (see Fig. 2), then the basin of attraction can be made arbitrarily close to the controllability domain [1]. In the case of the curvilinear beam, the boundary of the basin of attraction is unstable periodical cycle. It can be calculated solving equations of motion in the inverse time [2].

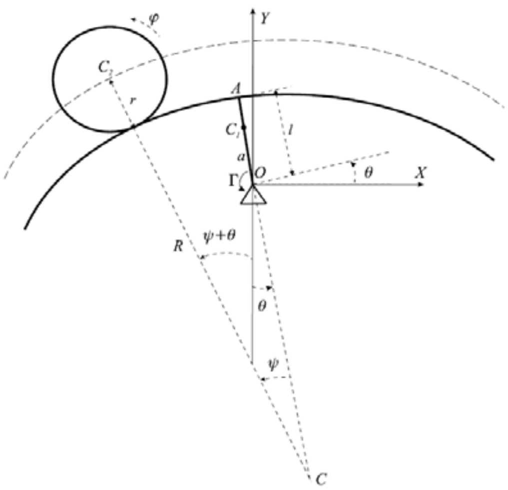

Figure 2.

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Optimal thrust programming along the brachistochronic trajectory with nonlinear drag

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The problem of maximization of the horizontal coordinate of mass-point moving in the vertical plane driven by gravity, viscous nonlinear drag, and thrust is considered. The slope angle and the thrust are considered as a control variables. The problem is related to the modified brachistochrone problem. Principle maximum procedure allows reducing the optimal control problem to the boundary value problems for three systems of two nonlinear differential equations. The extremal controls are designed in feedback form depending on the state variables. The qualitative analysis of the extremal trajectories is performed, and the characteristic properties of the optimal solutions are determined. It is shown that extremal thrust program involves a maximal boost at the beginning of the flight and ending with an intermediate thrust period.

Numerical study of a assistive strategy for human with passive exoskeleton

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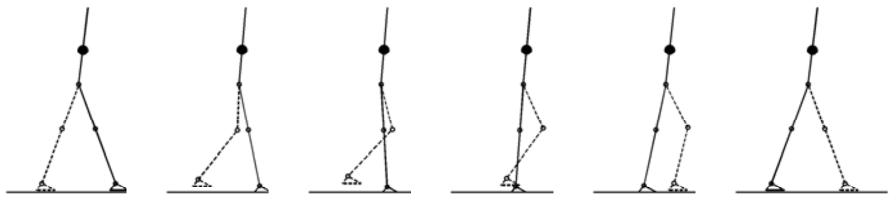
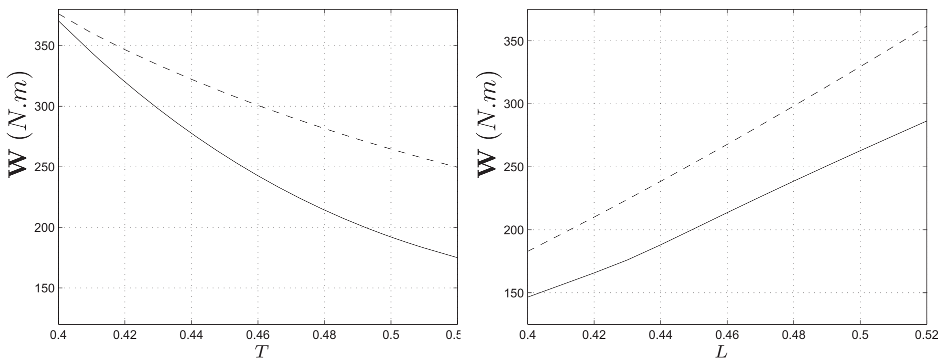
Abstract

The paper aim is to show theoretically the feasibility and efficiency of a passive exoskeleton for human walking and carrying a load. Human is modeled using a planar bipedal anthropomorphic mechanism. This mechanism consists of a trunk and two identical legs; each leg consists of a thigh, shin, and foot (massless). The exoskeleton is considered also as an anthropomorphic mechanism. The shape and the degrees of freedom of the exoskeleton are identical to the biped (to human). If the biped is equipped with an exoskeleton, then the links of this exoskeleton are attached to the corresponding links of the biped and the corresponding hip-, knee-, and ankle-joints coincide. We compare the walking gaits of a biped alone and of a biped equipped with exoskeleton (Fig. 1); for both cases the same load is transported. The problem is studied in the framework of ballistic walking model. During the ballistic walking of the biped with exoskeleton the knee of the support leg is locked, but the knee of the swing leg is unlocked. The locking and unlocking can be realized in the knees of the exoskeleton by any mechanical brake devices without energy consumption. There are not any actuators in the exoskeleton. Therefore, we call it *passive* exoskeleton. The walking of the biped consists of alternating single- and double-support phases. In our study, the double-support phase is assumed as instantaneous. At the instant of this phase, the knee of the previous swing leg is locked and the knee of the previous support leg is unlocked. Numerical results show that during the load transport the human with the exoskeleton spends less energy than human alone (Fig. 2). One of our perspectives is to investigate the case of a passive walking gait with single support and finite time double support phases.



$t = 0\ s$ $t = 0.09\ s$ $t = 0.18\ s$ $t = 0.27\ s$ $t = 0.36\ s$ $t = 0.45\ s$

Figure 1: Biped with exoskeleton, walking ballistic gait as a sequence of stick figures.



a) b)

Figure 2: a) W as function of T, $L = 0.5\ m$. b) W as function of L, $T = 0.45\ s$. Dashed lines for biped alone, solid lines for biped with exoskeleton.

The numerically-analytical approach to searching for limit cycles of an autonomous dynamical system with one degree of freedom

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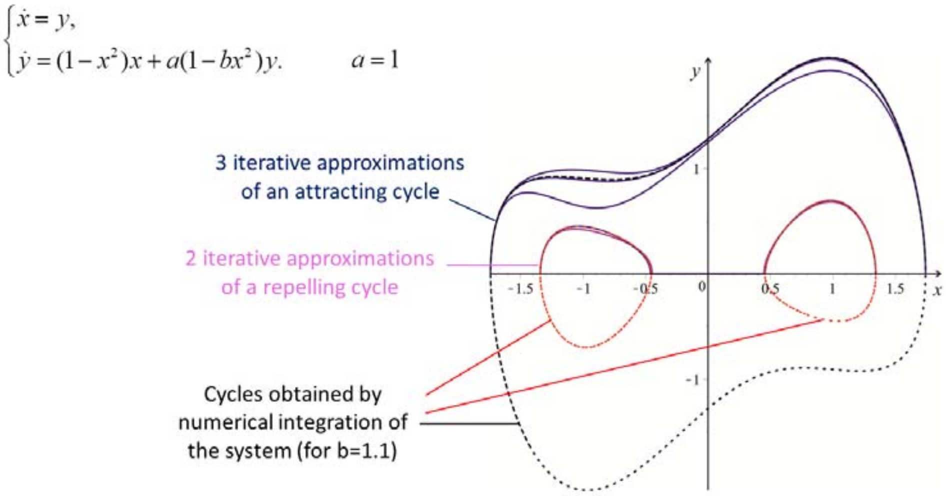
The numerically-analytical approach is proposed that provides iterative approximations of limit cycles of an autonomous dynamical system with one degree of freedom. At each iterative step, the averaging along the previous approximation curve is performed and the formal criterion is applied to ensure that the next approximation curve is closed. This criterion formally coincides with the Pontryagin criterion of emergence of limit cycles in a near-Hamiltonian system. However, for the new approach, the system can contain no any small parameter. Conditions of convergence of the iterative procedure are discussed. The proposed approach is closely related with the Samoilenko numerically-analytical method. The main advantage of the new approach is that it doesn't require a transition to a non-autonomous system. As an illustrative example, the method is applied to the perturbed Duffing oscillator.



Studying families of symmetric periodic solutions of Hill problem and its generalization*

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We consider nonintegrable Hamiltonian system with two degrees of freedom, namely celestial mechanics *Hill problem*, as a singular perturbation of an integrable one with quadratic Hamiltonian. Hill problem describes the motion of a massless body near the minor of two active masses and it is widely used in celestial mechanics and cosmodynamics.

Technique of generating solutions (see [1, 2] for the case of the restricted three-body problem) is applied for studying the families of symmetric periodic orbits. Each one-parameter family of such solutions is described in terms of a sequence of so called *arc-solutions* conjugated to each other over the certain rules by hyperbolic conics. The arc-solutions are such solutions of the integrable unperturbed problem that start and finish at the origin – the singular point of the perturbation function. Such approach allows describing not only periodic orbits but any invariant structure of the dynamical system that can be continued up to the limiting integrable problem as well.

Using generating solution it is possible to predict such properties of corresponding family of periodic orbits as a type of symmetry, global multiplicity of the orbit of generated solution and first approximation of the initial conditions and the period of the solution. An algorithm for investigation of families of symmetric periodic solutions over its generating sequence was proposed in [3]. This algorithm is applied to finding families of symmetric periodic obits of Hill problem. More than fifty new families of periodic solutions with different types of symmetry were found out and completely investigated [4]. The symmetry of generating solution plays an essential role for obtaining the initial condition of periodic orbit of the family. For computation of the whole family from one periodic solution a kind of predictor-corrector method is applied, which essentially explores the structure of the monodromy matrix of the periodic solution and provides the monitoring of bifurcations of the family.

Some generalization of the original Hill problem, which includes the singular perturbation with the opposite sign, was considered as well. Hamiltonian function of the generalized Hill problem takes the following form

$$\widetilde{H} = \frac{1}{2}\left(y_1^2 + y_2^2\right) + x_2 y_1 - x_1 y_2 - x_1^2 + \frac{1}{2}x_2^2 + \frac{\sigma}{r},$$

where $r = \sqrt{x_1^2 + x^2}$ and $\sigma = \pm 1$. For value $\sigma = -1$ one gets the Hamiltonian of the classical case of Hill problem. We call the problem with $\sigma = +1$ as *anti-Hill problem*. For value $\sigma = 0$ one gets so called integrable *Hénon problem* (known as Clohessy–Wiltshire equations) which particular solutions are used for construction generating sequences of families of periodic orbits of the generalized problem. The structure of families of periodic solutions of anti-Hill problem is considerably simpler than in the case of Hill problem and could be totally described with their generating solutions.

Intensive numerical computations allow to state that all known families of periodic solutions of Hill problem can be continued into the families of periodic solutions of anti-Hill problem but not vice verse, i. e. there are some anti-Hill problem's families that cannot be continued into Hill problem ones. More over, the further numerical experiment demonstrated that all families of Hill and anti-Hill problems form the common network connecting to each other by common generating solutions and by sharing common orbits with integer multiplicity of different families as well [5].

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Global bifurcation diagrams of a prescribed curvature problem arising in electrostatic micro-electro-mechanical systems

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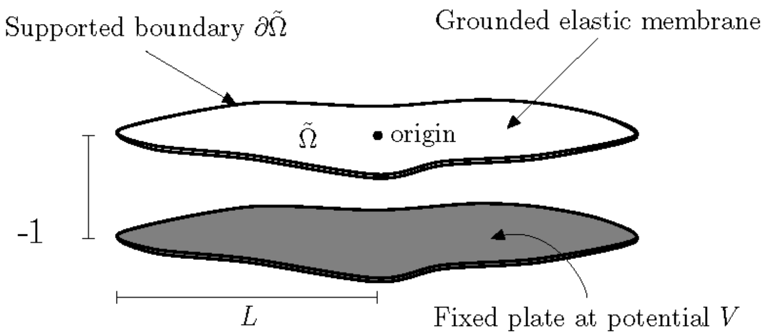
Abstract

We study global bifurcation diagrams and exact multiplicity of positive solutions for the one-dimensional prescribed curvature problem, which can be written in the equivalent dynamical system

$$\begin{cases} \dot{u} = v, \\ \dot{v} = -\lambda \dfrac{(1+v^2)^{3/2}}{(1-u)^p}, \end{cases}$$

where $0 \leq u < 1$, $\lambda > 0$ is a bifurcation parameter, and $p > 0$ is an evolution parameter. The problem is a derived variant of a canonical model used in the modeling of electrostatic Micro-Electro Mechanical Systems (MEMS) device obeying the electrostatic Coulomb law with the Coulomb force satisfying the inverse square law with respect to the distance of the two charged objects, which is a function of the deformation variable. The modeling of electrostatic MEMS device consists of a thin dielectric elastic membrane with boundary supported at 0 above a rigid plate located at -1.



Bifurcation diagram of self-sustained oscillations of an aerodynamic pendulum

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The dynamical model of the aerodynamic pendulum is constructed. This model is reduced to the perturbed Duffing equations. The following bifurcation is described: the asymptotically stable equilibrium (for which the pendulum is aligned along the flow) becomes unstable and two additional equilibria emerges alongside it (sidewise equilibria of the pendulum). It is known, that such bifurcation can be accompanied be the occurrence of periodic cycles (such cycles correspond to self-sustained oscillations of the pendulum). The bifurcation diagram describing amplitudes of these cycles depending on the bifurcation parameters is constructed using the Poincare-Pontryagin approach and related methods.

Dynamics of electronic neurons coupled via memristive device

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Biological neurons are coupled unidirectionally through a special junction, called a synapse. Electrical signal travels along a neuron after some biochemical reactions initiates a chemical release to activate an adjacent neuron. These junctions are crucial to cognitive functions, such as perception, learning and memory.

Thus, understanding neuron connections is a great challenge, which is needed to solve many important problems in neurobiology and neuroengineering for recreation of brain functions and efficient biorobotics. In this context, the ability of a memristive device to change conductivity under the action of pulsed signals makes it an almost ideal electronic analogue of a synapse. Memristive device represents a physical model of a Chua`s memristor, which is an element of electric circuits capable of changing the resistance depending on the electric signal received at the input.

In this work the memristive nanostructures Au/ZrO2(Y)/TiN/Ti was obtained by magnetron sputtering on oxidized silicon substrates. The thickness of the working dielectric (ZrO2(Y)) was 40 nm, the thickness of the top Au electrode was 40 nm, and the thickness of the bottom electrode TiN and Ti layers was 25 nm each. Such structure demonstrates reproducible switching between the low resistance state (LRS) and the high resistance state (HRS). Resistive switching is determined by the oxidation and recovery of segments of conducting channels (filaments) in the oxide film when voltage with different polarity is applied to it (accordingly RESET and SET current changes). The nonlinear dynamics of two electronic oscillators coupled via a memristive device has been studied. Such model implemented the interaction between synaptically coupled brain neurons with the memristive device imitating neuron axon. The synaptic connection is provided by the adaptive behavior of memristive device that changes its resistance under the action of spike-like activity. Dependences of the change in resistance on the frequency of the pulse signals are obtained, the different frequency-locking regimes have been studied in the frequency diagrams of the post-synaptic neuron in the space of the control parameters Mathematical model of such a memristive interface has been developed to describe and predict the experimentally observed regularities of forced synchronization of neuron-like oscillators.

This research may have the outmost importance in future research on synaptic plasticity (e.g., where the connections itselves are dependent on the dynamics), this can be a first step towards the simulation of the memory in living brain networks and creation of biologically plausible neuromorphic systems.

Acknowledgements

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