Dynamical Systems Methods in Fluid Mechanics

Organizers: Daniel Karrasch   Kathrin Padberg-Gehle

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Session 1: Coherence in nonautonomous fluid flows

Speaker: George Haller, ETH Zurich, Switzerland

Title: Invariant manifolds as barriers to stochastic and diffusive transport

Abstract: Observations of tracer transport in fluids generally reveal highly complex patterns shaped by an intricate network of transport barriers. The elements of this network appear to be universal for small diffusivities, independent of the tracer and its initial distribution. In this talk, I discuss a mathematical theory for weakly diffusive tracers that predicts transport barriers and enhancers solely from the flow velocity, without reliance on diffusive simulations. The theory also extends to particle motion under uncertainties, eliminating the need for Monte-Carlo simulations in detecting stochastic transport barriers. I illustrate the results on Rayleigh-Bénard convection simulation and on satellite-inferred ocean current data.

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Speaker: Christiane Schneide, Leuphana University Lüneburg, Germany

Title: Using a network ansatz to detect coherent sets in turbulent flows

Abstract: A coherent set in a non-autonomous dynamical system can be defined as a group of Lagrangian trajectories that has little interaction with other trajectories outside the set during a given time interval. This notion describes for instance a patch of warm water moving in cold water which disperses on a long time-scale. The identification of coherent behavior from given Lagrangian trajectory data is a subject of current research. Our approach uses a network representation of the Lagrangian data set. To this end, we interpret each Lagrangian trajectory as node of a network and create links between nodes depending on a distance measure between trajectories. Sets of different dynamical behavior within the flow can be made visible by analyzing classical network measures such as the node degree or by using spectral properties of the graph Laplacian matrix. We apply this method to three-dimensional flows in turbulent convection and compare the results of our Lagrangian analysis to Eulerian observations.

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Speaker: Péter Koltai, Free University Berlin, Germany

Title: From large deviations to semidistances of transport and mixing: coherence analysis for finite Lagrangian data

Abstract: In a quantitative, set-oriented approach to transport, finite time coherent sets play an important role. These are time-parametrized families of sets with unlikely transport to and from their surroundings under small random perturbations of the dynamics. Here we propose, as a measure of transport and mixing, (semi)distances that arise under vanishing perturbations in the sense of large deviations. Analogously, for given finite Lagrangian trajectory data we derive a discrete-time and space semidistance that comes from the „best" approximation of the randomly perturbed process conditioned on this limited information on the deterministic flow. It can be computed as shortest path in a graph with time-dependent weights. Furthermore, we argue that coherent sets are regions of maximal farness in terms of mixing, hence they occur as extremal regions on a spanning structure of the state space under this semidistance---in fact, under any distance measure arising from the physical notion of transport and mixing. Based on this notion we develop a tool to analyze the state space (or the finite trajectory data at hand) and identify coherent regions. We validate our approach on idealized prototypical examples and well-studied standard cases. This is joint work with Michiel Renger.

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Speaker: Daniel Karrasch, Technical University Munich, Germany

Title: How does advection-diffusion look like in Lagrangian coordinates?

Abstract: The aim of this talk is twofold. First, I discuss the advection-diffusion equation in a Lagrangian, i.e., material-based frame, and specifically the dual relation between the diffusivity (tensor field) and an induced Riemannian geometry. Second, I discuss relations to the diffusion-map approach to isometric manifold embedding. Several published methods for data-based Lagrangian coherent structure detection are re-interpreted within this framework. By analogy to isometric manifold embeddings, we transform the dynamical advection–diffusion process into a static picture, which then answers the question in the title. This is joint work with Álvaro de Diego (TU München) and Péter Koltai (FU Berlin).

Session 2: PDE and multi-scale methods

Speaker: Camilla Nobili, University Hamburg, Germany

Title: Limitation of the background field method for the Rayleigh-Bénard convection model

Abstract: We consider the Rayleigh-Bénard convection as modeled by the Boussinesq equations, in case of infinite Prandtl number. In several works, the background field method applied to the temperature field has been used to provide upper bounds on average upward heat flux (the Nusselt number) in terms of the Rayleigh number. In these applications, the background field method

comes in form of a variational problem where one optimizes a stratified temperature profile subject to a certain stability condition; the method is believed to capture marginal stability of the boundary layer. In this talk we demonstrate the limitation of this method in reproducing physical bounds on the Nusselt number and we briefly discuss some new perspective.

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Speaker: Michael Dellnitz, University Paderborn, Germany

Title: Glimpse of the Infinite – on the Approximation of the Global Dynamical Behavior for Partial Differential Equations

Abstract: Over the last years so-called *set oriented* numerical methods have been developed for the analysis of the long-term behavior of finite-dimensional dynamical systems. The underlying idea is to approximate the corresponding objects of interest—for instance *global attractors* or related *invariant measures*—by box coverings which are created via multilevel subdivision techniques. That is, these techniques rely on partitions of the (finite-dimensional) state space, and it is not obvious how to extend them to the situation where the underlying dynamical system is infinite-dimensional. In this talk we will present a novel numerical framework for the computation of finite-dimensional dynamical objects for infinite-dimensional dynamical systems. Within this framework we will extend the classical set oriented numerical schemes mentioned above to the infinite-dimensional context. The underlying idea is to utilize appropriate *embedding techniques* for the reconstruction of global attractors in a certain finite-dimensional space. We will apply our approach to partial differential equations and problems in fluid mechanics.

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Speaker: Christian Kuehn, Technical University Munich, Germany

Title: Stability Exchange in Dynamics with Applications to Fluids

Abstract: In this talk, I shall start to explain the basic principle of delayed orbits near bifurcations points and their classical use in multiple time scale dynamical systems. Then I am going to proceed to outline two possible extensions, which are recently motivated by two application areas: (I) random dynamical systems with uncertain parameters and initial data, and (II) particle-surface interaction in fluid dynamics. Both extensions hint at the fact that the principle of exchanging stability can be very subtle in complex dynamical systems in comparison to the classical low-dimensional situation. The details of the talk can be found in the two works (I) "Uncertainty transformation via Hopf bifurcation in fast-slow systems", C. Kuehn, Proceedings of the Royal Society A, Vol. 473, 20160346, 2017 and (II) "Tracking particles in flows near invariant manifolds via balance functions", C. Kuehn, F. Romano and H. Kuhlmann, Nonlinear Dynamics, 2018.

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Speaker: Marcel Oliver, Jacobs University Bremen, Germany

Title: Optimal balance and fast-slow splittings in geophysical fluid dynamics

Abstract: The dynamics of linear rotating shallow water flow splits into (slow) Rossby waves and (fast) gravity waves. Under the full nonlinear dynamics, this splitting persists over long time through adiabatic invariance of the energy in the fast degree of freedom. Moreover, the fast energy is adiabatically invariant under slow non-autonomous deformation of the nonlinear interactions. This observation leads to a computational procedure, in this context introduced by Viudez and Dritschel (2004) under the name "optimal balance", to compute the fast-slow splitting of a nontrivial flow by adiabatically deforming the system from a linear regime where the splitting is explicit and exact to the fully nonlinear regime; this leads to the requirement to solve a boundary value problem in the time domain. We show, in an idealized setting, that the resulting splitting is almost optimal in a precise sense, discuss the efficient implementation of the boundary value solver, and look at the implementation for the shallow water equations in primitive variables.

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Speaker: Makoto Iima, Hiroshima University, Japan

Title: A Jacobian-free algorithm to calculate the phase sensitivity function of the phase reduction theory and its application to Kármán's vortex street

Abstract: The phase reduction theory describes the dynamics in the neighborhood of the limit cycle by a single variable called “phase”. The phase sensitivity function, one of the basic quantities in the phase reduction theory, has been calculated by numerical integration of the adjoint equation that includes the transpose of the Jacobian of the system. When applying the phase reduction theory to the fluid systems, we have problems to overcome, that is, (1) even in the numerical calculation, the system is described by a large number of degrees of freedom, (2) the Jacobian is not obtained explicitly. In this talk, we propose a method to save computational resources so that the phase reduction theory is applicable for various fluid systems. Using this method, we can calculate the phase sensitive function without explicit expression of the Jacobian, and computation time can be reduced compared to the direct integration of the adjoint equation. We will report the application to a one-dimensional traveling pulse solution of a reaction-diffusion system under periodic boundary condition as well as the two-dimensional Kármán's vortex street.

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