Nonlinear Schrödinger Models and Rogue Waves: Minisymposium at Dynamics Days Europe 2018

Organisers:
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Current list of mini-symposia is available at http://dynamicsday2018.lboro.ac.uk/

1 MS Description

Nonlinear Schrödinger equation and its generalisations play fundamental role in modern nonlinear physics and mathematics. The mini-symposium will bring together an international collection of mathematicians and physicists in order to identify common interests and emerging problems involving nonlinear Schrödinger models and their applications to the description of modulational instability, dispersive shock and rogue waves, integrable turbulence and related phenomena. The topics of interest include the latest developments in the inverse scattering transform techniques, semi-classical asymptotics, integrable turbulence, rogue waves, numerical simulations and physical experiments in water waves and nonlinear optics.

2 Schedule

All 25-min talks are in room J002 Edward Herbert Building

Thursday $10:30-12:30$

Roger Grimshaw (Department of Mathematics, University College London, UK)
Sara Lombardo (Loughbourough University, UK)
Alexander Tovbis (University of Central Florida, USA)
Matteo Conforti (Univ. Lille, CNRS, UMR 8523-PhLAM- F-59000 Lille, France

Thursday $15:30-18:00$

Pierre Suret (University of Lille, France)
Alfred Osborne (Nonlinear Waves Research Corporation, Alexandria Virginia, USA)
Andrey Gelash (Novosibirsk State University)
Dmitry Agafontsev (P.P. Shirshov Institute of Oceanology, Moscow)
Bertrand Kibler (Laboratoire Interdisciplinaire Carnot de Bourgogne, Dijon, France)

Friday $10:30-12:00$

Amin Chabchoub (University of Sydney)
Bob Jenkins (Department of Mathematics, University of Arizona, USA)
Ragnar Fleischmann (Max-Planck-Institut, Gẗtingen, Germany)

3 Abstracts

Thursday $10:30-12:30$

1. Roger Grimshaw (Department of Mathematics, University College London, UK)

   The effect of wind on modulational instability

   Various mechanisms have been proposed to describe the generation of water waves by wind. But despite decades of theoretical research, observations and more recently, detailed numerical simulations, the nature of these mechanisms and their practical applicability remains controversial. Two main mechanisms are currently considered. One is a classical shear flow instability mechanism originally developed by Miles (1957) and subsequently adapted for routine use in wave forecasting models. The other is a steady- state theory, developed originally by Jeffreys (1925) for separated flow over large amplitude waves, and later adapted by Belcher and Hunt (1998 review article) for non-separated flow over low-amplitude waves. Neither theory alone has been found completely satisfactory, and in particular, both fail to take account of wave transience and the tendency of water waves to develop into wave groups. In this talk we revisit the Miles mechanism, but explicitly take account that the wave field has a group structure. The main outcome is that as well as a complex-valued phase speed, we find that there is a small complex-valued component to the wavenumber, which leads to an enhanced growth rate in the reference frame moving with the wave group velocity. This linear theory is then developed into a forced nonlinear Schrödinger equation. We show that the instability persists into this weakly nonlinear regime, and that modulationall instability is enhanced with superexponential growth.

2. Sara Lombardo (Loughbourough University, UK)

   Nonlinear waves instabilities by means of elementary algebraic-geometry: NLS-type models

   In this talk I will consider the linear stability properties of NLS-type systems and their continuous wave solutions. I will revisit the scalar NLS equation in this context and consider then the integrable coupling of two nonlinear Schroedinger equations. Using the integrable properties of the system, one can compute and classify the so-called stability spectra, providing a necessary condition in the parameters space for the onset of instability. The derivation of the spectra is completely algorithmic and make use of elementary algebraic-geometry. It turns out indeed that, for a Lax Pair that is polynomial in the spectral parameter, the problem of classifying the spectra is transformed into a problem of classification of certain algebraic varieties.

   The method is general enough to be applicable to a large class of integrable systems.

   This work has been done in collaboration with Antonio Degasperis (Roma "Sapienza") and Matteo Sommacal (Northumbria).

3. Alexander Tovbis (University of Central Florida, USA)

   Towards kinetic equation for soliton and breather gases for the focusing Nonlinear Schroedinger equation

   By A. Tovbis and G. El

   Abstract: Kinetic equation for a soliton gas for the Korteweg - de Vries equation was first proposed by V. Zakharov and later derived by G. El using the thermodynamic limit of the KdV-Whitham equations. Later, G. El and A. Kamchatnov proposed kinetic equation for the soliton gas for the focusing Nonlinear Schroedinger (fNLS) equation using physical reasoning.

   In this talk, we consider the large N limit of nonlinear N-phase wave solutions to the fNLS equation subject to a certain scaling of the corresponding bands and gaps. In this limit, we obtain integral equations for the scaled wavenumbers and periods and, as a consequence, derive the kinetic equation for soliton and breather gases, which takes into account soliton-soliton and soliton-background interactions. A number of particular examples is considered. Our approach can be used to derive kinetic equation for the soliton gas on the background of any finite gap solution.

4. Matteo Conforti (Univ. Lille, CNRS, UMR 8523-PhLAM-Physique des Lasers Atomes et Molécules, F-59000 Lille, France

   Dispersive decay of discontinuous pulses in optical fibers

   A. Bendahmane, G. Xu, A. Kudlinski, A. Mussot, M. Conforti (Lille)

   S. Trillo Department of Engineering, University of Ferrara, Via Saragat 1, 44122 Ferrara, Italy

   We study the decay of a discontinuous pulse occurring during the propagation in an optical fiber, in the normal dispersion and highly nonlinear regime. We consider step-like initial conditions for the power or the frequency chirp of the pulses, which correspond to the fluid height and velocity of the hydrodynamic analogy with the shallow water (Saint-Venant) equations, valid in the limit of vanishing dispersion. In the dispersionless limit, the step decays into a wave pair, consisting either of rarefaction or shock type connected by a constant state. The presence of dispersion regularizes the shock through fast oscillation, generating a dispersive shock wave. In this regime, the Whitham modulation theory can be applied to predict the different wave pairs configurations, separated by constant or oscillating states. We study the different regimes and characterize the phase transitions between them by means of a fiber optics experiment. As relevant examples, we consider typical problems of fluid dynamics such as the dam breaking or the piston problem, and the generation of cavitating states. The experiment also constitutes, to the best of our knowledge, the first fully quantitative test of Whitham theory.

Thursday $15:30-18:00$

5. Pierre Suret (University of Lille, France)

   Integrable turbulence in the focusing regime of 1D nonlinear Schrodinger equation : optical experiments and semiclassical theory

   By P. Suret, A. Tikan, R. El Koussaifi, S. Bielawski, C. Szwaj, C. Evain, G. El, S. Randoux

   The field of nonlinear fibres optics is a promising tabletop laboratory for the experimental investigation of the physics of 1D nonlinear Schrodinger Equation (1DNLSE) and in particular of integrable turbulence phenomena [1-4]. The very recent advances of the ultrafast measurement techniques (time lens) have allowed the measurement of the dynamics both of the intensity [3] and of the phase [4] on sub-picosecond timescales. We will review the experimental results on the statistical, spectral and dynamical properties of nonlinear random waves propagting in optcial fibers.

   We will also review the existing theoretical approaches and open questions. We will show in particular that some results obtained in the semi-classical limit [5,6] provide clear interpretation of the experimental observations such as the emergence of structures locally very close to the Peregrine soliton [3-6].

   References

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6. Alfred Osborne (Nonlinear Waves Research Corporation, Alexandria Virginia, USA)

   Breather turbulence in ocean surface waves

   The study of nonlinear effects in ocean surface waves has a long history and usually focuses on the Stokes wave nonlinearity. However, modern theories of wave motion also include the dynamics of the nonlinear Schroedinger equation (NLS), a complex nonlinear wave equation of mathematical physics that has many spectral solutions, including Stokes waves and breathers as nonlinear components. Here I use the NLS equation as a method of nonlinear Fourier analysis and I project measured oceanic time series onto the modes of the NLS equation. In may cases, such as in Currituck Sound and in the North Sea, I find a large number of breather trains in each time series, typically near the peak of a storm. The number of the breathers is typically 200 for a half-hour time series. I discuss methods for interpreting these energetic sea states in terms of breather turbulence, a paradigm of the integrable structure of the NLS equation as described by finite gap theory. The study of rogue or freak waves depends crucially on knowledge of breather dynamics and I discuss these issues from both theoretical and experimental points of view.

7. Andrey Gelash (Novosibirsk State University)

   Dynamics and statistics of solitons and breathers in the nonlinear Schrodinger equation: from infinite line to periodic boundary conditions

   We present our recent results on the theory of the focusing Nonlinear Schrodinger Equation (NLSE) with infinite line boundary conditions [1-3] and discuss their applications at the periodic boundary conditions (see, for instance, [4-6]). First, we find exact formulas for the space-phase shifts that NLSE breathers acquire after collisions [1]. These formulas allow us to construct different scenarios of the multi-breather interactions, in which the breathers collide with each other at certain phases producing rogue waves or small perturbations of a background. In particular, we show how the rogue waves arise from the local condensate perturbations formed by the NLSE breathers. We present new scenarios of rogue wave formation for randomly distributed breathers as well as for artificially prepared initial conditions. Then we present new numerical approaches to solving the inverse scattering problem for the NLSE. We show how to generate multi-soliton and multi-breather solutions with the number of solitons (or breathers) more than 100, i.e. when the inverse scattering transform formulas are ill-conditioned [2]. In addition, we present new efficient numerical approaches to solving the inverse scattering problem in pure non-solitonic case [3] and when solitons coexist with incoherent waves. These numerical approaches can be used to study the problem of integrable turbulence [2]. Also we put the localized solutions of the NLSE under periodic boundary conditions assuming that the wave system period is large in comparison with a characteristic size the solutions. In this case the inverse scattering transform spectrum of the solutions (scattering data) changes. The discrete eigenvalues corresponding to solitons and breathers transform to narrow eigenvalue gaps. We show the evolution of large ensembles of solitons and breathers under periodic boundary conditions and thereby pave the way to establish a connection between the infinite line and periodic IST approaches. Finally, we demonstrate how solitons and breathers can be identified in complex periodic wave field by analysing its scattering data.

   References

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8. Dmitry Agafontsev (P.P. Shirshov Institute of Oceanology, Moscow)

   Strongly interacting soliton gas and formation of rogue waves

   co-author : A.A. Gelash

   We study numerically the properties of (statistically) homogeneous soliton gas depending on soliton density (proportional to number of solitons per unit length) and soliton velocities, in the framework of the focusing one-dimensional Nonlinear Schrodinger (NLS) equation. In order to model such soliton gas we use N-soliton solutions (N-SS) with N 100, which we generate with specific implementation of the dressing method combined with 100-digits arithmetics. We examine the major statistical characteristics, in particular the kinetic and potential energies, the kurtosis and the PDF of wave intensity.

   We show that in the case of small soliton density the kinetic and potential energies, as well as the kurtosis, are very well described by the analytical relations derived without taking into account soliton interactions. With increasing soliton density and velocities, soliton interactions enhance, and we observe increasing deviations from these relations leading to increased absolute values for all of these three characteristics. For rarefied soliton gas, the PDF of wave intensity deviates from the exponential PDF drastically, transforming much closer to it at densities corresponding to essential interaction between the solitons. Rogue waves emerging in soliton gas are multi-soliton collisions, and yet some of them have spatial profiles very similar to those of the Peregrine solutions of different orders. We present examples of two- and three-soliton collisions, for which even the temporal behavior of the maximal amplitude is well-approximated by the Peregrine solutions.

9. Bertrand Kibler (Laboratoire Interdisciplinaire Carnot de Bourgogne, Dijon, France)

   Modulation instability, breathers and frequency combs in optical fibers

   Since the seminal works by A. Hasegawa and co-workers in the 1980s, the modulation instability phenomenon has been widely studied and used in optical fibers, in particular for generating high-repetition-rate soliton trains and for parametric amplification of weak signals. Modulation instability is also known as a general precursor of highly localized wave structures through amplification of perturbations. We review here our recent experiments performed in nearly-conservative physical systems based on nonlinear fiber optics and using the coherent seeding of the modulation instability process. We evidence a large class of exact breather solutions of the nonlinear Schrdinger equation. These results have shed new lights on extreme nonlinear dynamics and related analogies between optics and hydrodynamics, and could have potential applications in the development of optical frequency combs.

Friday $10:30-12:00$

10. Amin Chabchoub (University of Sydney)

    Hydrodynamic wave shoaling and dispersive shock waves

    O.Kimmoun, H. Hsu, S, Trillo and A. Chabchoub

    Hydrodynamic dispersive shock waves (DSW) are usually observed in shallow water regime and describe the dynamics of undular or tidal bores. DSW can be also observed in various nonlinear dispersive media. Indeed, the experimental observations of such waves have been reported first in the 70s in plasma. Recently, Fatome et al. (2014) have experimentally confirmed that the propagation of optical pulses can lead to multiple optical dispersive shock events that interact as a result of four-wave mixing. We report the an experimental study that has been performed in a large 200 m wave flume with a gradually varying bathymetry that consists with two flat parts connected by a 1/20 slope. We show that when breathers propagates over such type of bottom, DSW dynamics can be observed while the nonlinear patterns measured are similar to the the optical observations described in Fatome et al. (2014).

11. Bob Jenkins (Department of Mathematics, University of Arizona, USA)

    Long time behavior of the derivative nonlinear Schrödinger equation

    Abstract: In this talk I will discuss our recent work on the derivative nonlinear Schrdinger (DNLS) using the inverse scattering transform (IST). I will briefly discuss our new well-posedness results that utilize the inverse scattering transform. Then I will describe how the IST can be used to derive rigorous point-wise asymptotic behavior for the solutions of the Cauchy problem for DNLS on the line. Our result established soliton resolution for any generic initial data in $H^{2,2}$ functions with two derivatives and two moments in L^2. This is joint work with Jiaqi Liu, Peter Perry, and Catherine Sulem.

12. Ragnar Fleischmann (Max-Planck-Institut, Gẗtingen, Germany)

    Branched flows and extreme waves in weakly scattering random media

    Electron waves scattered by residual disorder in high mobility semiconductors, ocean waves deflected by eddies in the water currents, and sound waves scattered in the turbulent atmosphere. These are only a few examples of waves that in their natural environment are weakly scattered by a complex medium. Due to their internal structure the environments are best described as correlated random media. Between ballistic and diffusive behaviour these systems exhibit an additional transport regime characterized by heavy-tailed intensity distributions and drastically increased probability of extreme events: the regime of branched flows.

    Probably the most consequential occurrences of branched flows can be observed in the propagation of tsunamis. Ocean depth fluctuations of only a few percent lead to strong random focusing of the tsunami waves into branched-like patterns and easily to intensity variations of an order of magnitude, challenging reliable tsunami predictions [1]. The statistics of branched flows is closely connected to the formation of random caustics that show universal scaling behaviour [2, 3] and govern the statistics of extreme events [4]. First results show an intricate interplay of random caustics and nonlinear focusing in the branched flows of nonlinear Schrödinger equations.

    References

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